

Problem-Solving and Mathematical Reasoning

K-5 Module



Your Opinion



- What does it mean to do mathematics?
- How do you know if you are good at it?



Process Is Important

Process is as much a part of doing mathematics as the content itself



NCTM Standards

Content Standards

- Algebra
- Number Sense
- Geometry
- Measurement
- Data & Probability

Process Standards

- Communication
- Representation
- Problem-solving
- Reasoning and proof
- Connections



NC Curriculum Standards

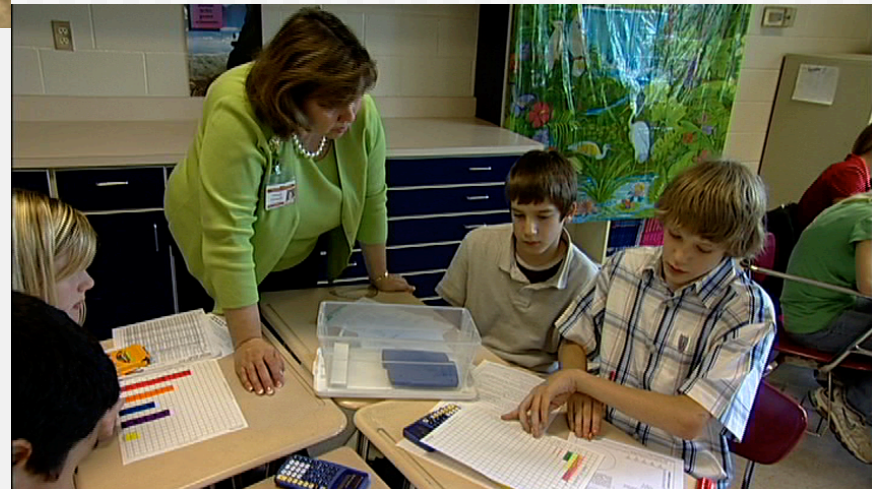
- New objectives for K-5
- Strand 6: Problem-Solving
The learner will solve problems and reason mathematically



More to do in the same amount of time?



No !



Number Talks

- A class conversation about an arithmetic problem, in which students discuss and critique various strategies for solving the problem
- Work is done mentally, though some writing may be offered by a student or by the teacher when a strategy is explained



Goals of Number Talks

- To let reason, not the teacher, be the authority in determining whether or not a strategy works
- To use mental arithmetic in developing numerical reasoning
- To learn basic facts through reasoning and discussion instead of isolated drill



Goals of Number Talks

- To increase fluency in operations with small numbers in order to increase fluency in operations with large numbers
- To provide a variety of strategies in order to increase both fluency and proficiency

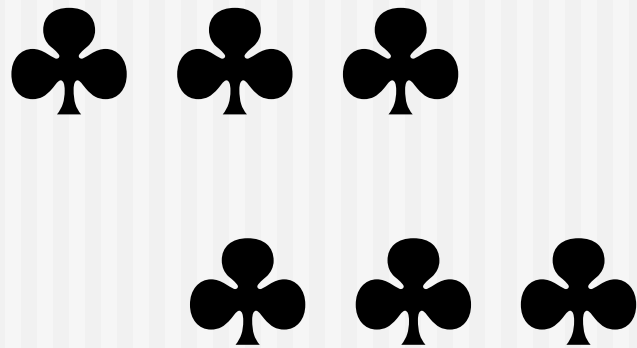


Directions: Number Talks

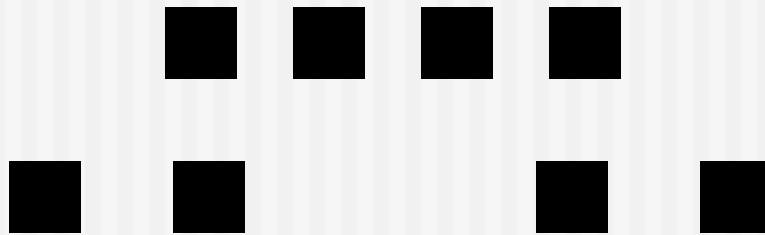
- 1) When the problem is put up, solve in your head
- 2) When you have solved, put your thumb up in front of your chest
- 3) Try to solve in a different way - for each different way you solve, put up another finger



How many objects do you see?



How many objects do you see?



Let's Do Some Addition

- $16 + 9 =$
- $17 + 13 =$
- $26 + 18 =$
- $291 + 55 =$



Writing Strings of Numbers

- Be careful when writing strings of numbers not to give the impression that the = sign means the answer is coming
- For example, it is not unusual to see

$$9 + 6 = 15 + 10 = 25$$

written on the board

Is this true?



Keep It Short!

Number talks should be ...

- Used frequently
- Be short (approximately 10-15 min.)
- Focused discussions on two or three problems



Procedural Fluency

Procedural fluency DOES NOT mean

- Speed

Procedural fluency DOES mean

- Accuracy
- Efficiency
- Flexibility



Relating to Process Standards

When you were working did you...

- Make connections among different content strands of mathematics?
- Use different representations?
- Communicate mathematical ideas?
- Reason and prove?
- Problem-solve?



I say...



CAT



Did you say...



DOG



I say...



mustard



Did you say...



ketchup



I say...

Problem-solving

What will you say?



Common Lesson Format in the US and Germany

- Teacher provides instruction about a concept or skill
- Teacher solves example problems with the class
- Students practice similar problems while teacher helps individual students

Martinez, J.G.R. (2001). Exploring, inventing, and discovering mathematics: A pedagogical response to the TIMSS. *Mathematics Teaching in the Middle School*. 7(2), 114-119.



Common Lesson Format in Japan

- Teacher poses a complex problem
- Students struggle with the problem
- Various students present ideas of solutions to the class and the strategies are discussed
- Teacher summarizes class' conclusions
- Students practice similar problems

Martinez, J.G.R. (2001). Exploring, inventing, and discovering mathematics: A pedagogical response to the TIMSS. *Mathematics Teaching in the Middle School*. 7(2), 114-119.



Implications

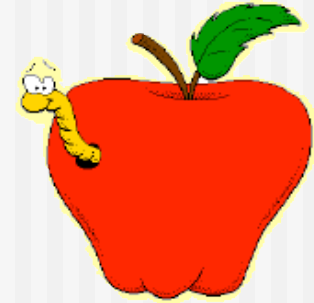
- What implications does reorganizing the traditional lesson format have for North Carolina teachers?
- How would the role of the teacher change?
- What implications does reorganizing the lesson format have for students?



Less Is More

- Present fewer tasks in more depth
- Choose engaging, challenging, and worthwhile tasks that address the relevant mathematics goals
- Make connections among different strands of mathematics and different representations





Classroom Climate

- Expect engagement
- Allow time for struggle
- Encourage risk-taking and persistence
- Value mistakes
- Create a sense of belonging
- Involve everyone in meaningful participation



Subtraction Number Talks

■ $21 - 6 =$

■ $30 - 14 =$

■ $64 - 35 =$

■ $462 - 16 =$



Math Talk

- Discussion in the classroom becomes a critical part of making sense of mathematics
- Teachers must become good listeners who pay close attention to student thinking and use that information to make “in the moment” and long range instructional decisions



Classroom Discussions

Read your section of the article by Kazemi & Hintz (2008). Be prepared to share with your group:

- Key points
- A favorite quote
- Questions you have



Choral Counting





Choral Counting

- Read the vignette
- How did the teacher...
 - Support students to know what to share and how to share?
 - Support students to be positioned competently?
 - Achieve a mathematical goal?



Multiplication Number Talk

■ $8 \times 5 =$

■ $18 \times 8 =$

■ $13 \times 11 =$

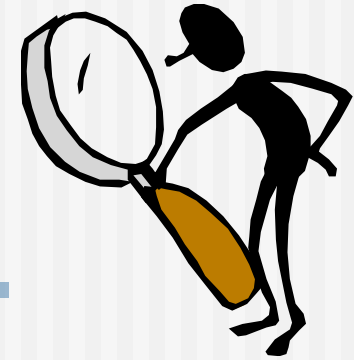
■ $4 \times 24 =$



Open-ended Tasks

- Take longer, but involve more math
- Represent a restructuring of time
- Usually need some introduction
- Afford teachers an opportunity to informally assess as students work
- Teachers can plan for follow-up discussions





Representations

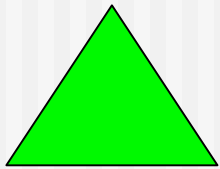
- Physical or pictorial pattern
- Chart or table
- Symbolic equation or expression
- Graph
- Language

Van de Walle, J.A. (2004). *Elementary and middle school mathematics: Teaching developmentally*. Pearson Learning, Inc.

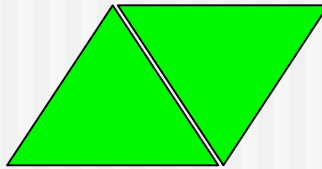


Perimeter Trains

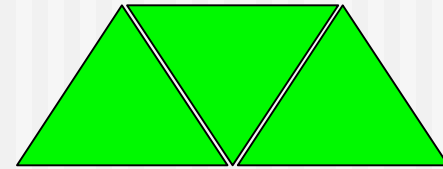
- Find the perimeter of the following trains



Train 1



Train 2



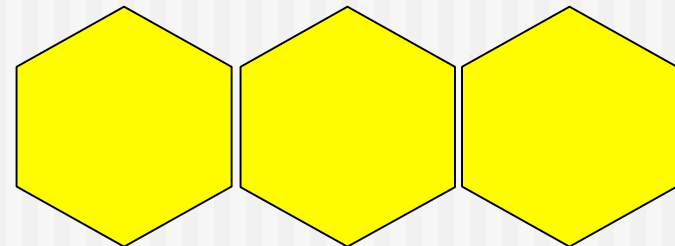
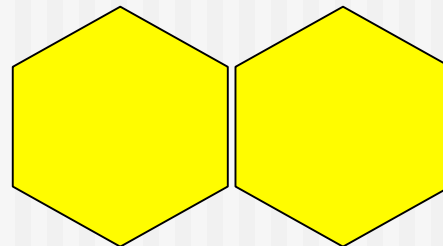
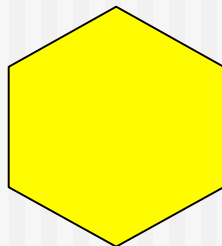
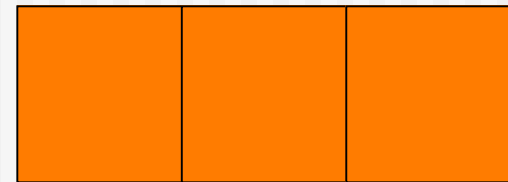
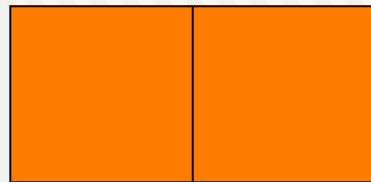
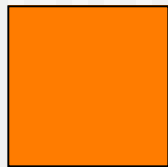
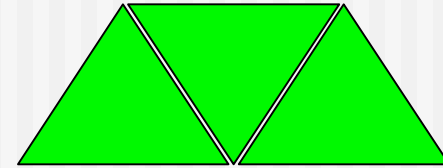
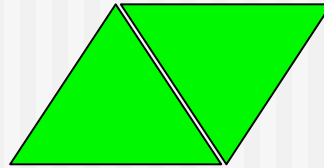
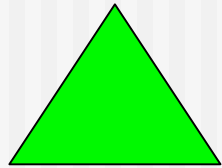
Train 3



Train 1

Train 2

Train 3



Sharing Solutions

Find a person who worked in a different group and share your work

- How did you approach the task?
- In what ways were your approaches the same? In what ways were they different?
- What questions do you both still have?



Representation

- Representations are necessary to students' understanding of mathematical concepts and relationships
- Representations allow students to communicate mathematical approaches, arguments, and understanding to themselves and to others
- They allow students to recognize connections among related concepts and apply mathematics to realistic problems

~PSSM, 2000



Let's Make a List

- What mathematics content did you use to work on this task?
- What mathematics processes did you use?



Making Connections

- What connections have we made among strands in doing this activity?
- Think about connections across grades, what was done in K-2 to build background for this activity?
- What in this activity is building a foundation for in upper level mathematics?



Analyzing Student Work

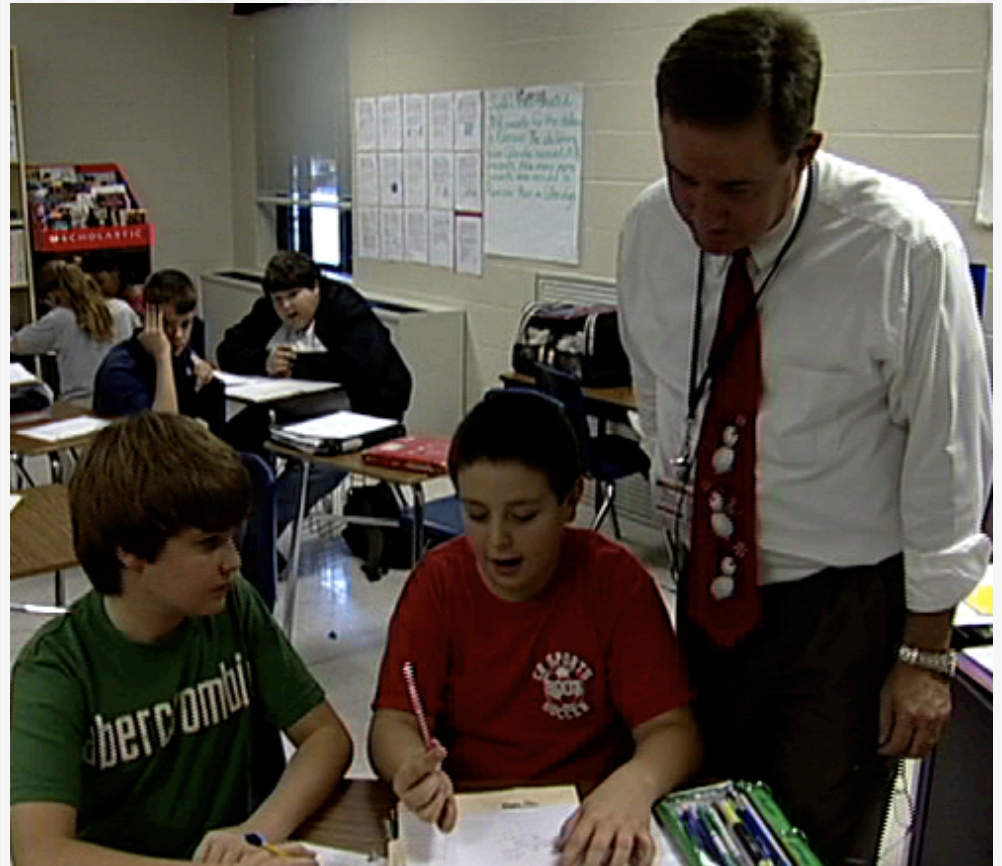
With your partners look at the sample student papers

- Which student's thinking do you feel you know the most about? Why?
- What do the students understand?



What Next?

If this set of papers represents the class as a whole, what lesson or task might you plan next?



Dare We Try Division?

■ $8 \div 2 =$

■ $40 \div 3 =$

■ $356 \div 5 =$

■ $356 \div 25 =$



Problem Solving Strategies

- What problem solving strategies should students be able to apply fluently?
- How can modeling these strategies become routine in your classroom?
- How are these strategies intertwined with process skills?



Basics for the 21st Century

- Our students will face a different kind of competition as workers
- Workers need mathematics content knowledge and confidence in themselves to use the mathematics in routine and creative ways
- Education must meet their needs



The 25% of the
population in China
with the **highest IQs** ...



is greater than the
total population of
North America



In India, it's the
top 28%



Translation for educators:

they have more honors
kids than we have kids



China will soon become
the number one
English-speaking
country in the world



If you took **every**
single job in the U.S.
today and shipped it
to China . . .



China still would
have a **labor surplus**



The U.S. Department of
Labor estimates that
today's learner will have
10 to 14 jobs
by age 38



According to the
U.S. Department
of Labor . . .



1 out of 4 workers today
is working for a
company for whom they
have been employed
less than 1 year



According to former
Secretary of Education
Richard Riley . . .



the top 10 jobs that
will be in demand in
2010 **didn't exist**
in 2004



We are currently
preparing students
for jobs that
don't yet exist . . .



using
technologies that
haven't yet been
invented ...



in order to solve
problems we don't
even know are
problems



What does it all mean?

Change is happening
faster than we ever
imagined



Basics for the 21st Century

- **WE** are preparing the workers for the 21st Century
- Basics include understanding of mathematical concepts, procedural fluency, strategies for solving problems, and ability to communicate thinking



Now that we know . . .

How are we teaching?



Reflection

- Research indicates that reflection is a powerful tool for learning - one that needs to be used frequently in every classroom
- Look back at Goal 6 of the NC Curriculum Standards and complete the reflection questions in the handout



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